四技二專聯合複習考試
共同考科　數學(C)卷　詳解

數學(C)卷 II002-C

1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	17	18	19	20	21	22	23	24	25
C	B	D	A	B	D	C	C	A	D	C	B	C	B	A	D	C	A	D	D	B	A	B	D	A

1. $\because a^2+b<0 \Rightarrow b<0$
   $\because 3ab>0$ and $b<0 \Rightarrow a<0$
   $\Rightarrow (a,b)$ 在第三象限

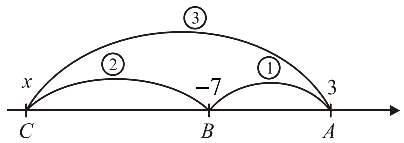
2. $\overline{AO}=\sqrt{3^2+7^2}=\sqrt{58}$
   $\overline{BO}=\sqrt{8^2+(-1)^2}=\sqrt{65}$
   $\overline{CO}=\sqrt{5^2+5^2}=\sqrt{50}$
   $\overline{DO}=\sqrt{4^2+6^2}=\sqrt{52}$
   $\Rightarrow B$ 點離原點最遠

3. $(\pi=180°)$，$\dfrac{\pi}{5}-20°=36°-20°=16°$
   $\dfrac{\pi}{6}-15°=30°-15°=15°$
   $\dfrac{\pi}{7}-10°=25.\cdots°-10°=15.\cdots°$
   $\dfrac{\pi}{8}-8°=22.5°-8°=14.5°$

4. $\because \overline{AB}=5 \Rightarrow |\,3-x\,|=5 \Rightarrow 3-x=5$ or -5
   $\Rightarrow x=-2$、$8 \Rightarrow -2+8=6$

5. $\because \cos\theta$ 的週期為 2π
   $\Rightarrow 7\cos(4x+5)+2$ 的週期為 $\dfrac{2\pi}{4}=\dfrac{\pi}{2}$

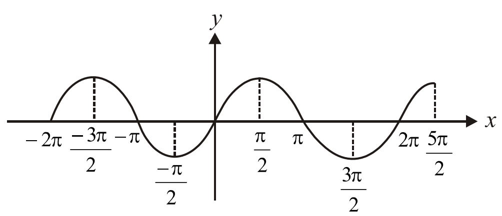
6. 如下圖 $\Rightarrow \overline{AB}$ ： $\overline{BC}=1$ ：2



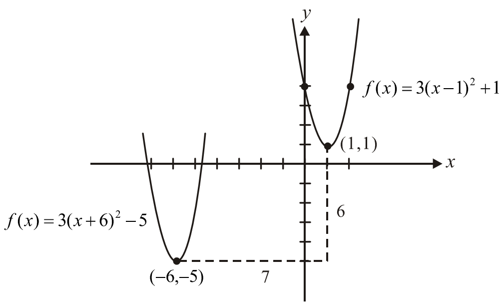
   利用內分點公式 $\Rightarrow -7=\dfrac{1\cdot x+2\cdot 3}{1+2}$
   $\Rightarrow -21=x+6 \Rightarrow x=-27$

7. $1230-360\times 8=-1650=a$
   $-3120+360\times 14=1920=b$
   $\Rightarrow a+b=-1650+1920=270$

8. 由圖可知 $\dfrac{\pi}{2}\le x\le \pi$
   $y=\sin x$ 為減函數



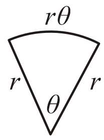
9. $f(x)=3(x+6)^2-5$，頂點為 $(-6,-5)$
   $f(x)=3(x-1)^2+1$，頂點為 $(1,1)$
   $\Rightarrow (-6,-5)$ 向 x 軸正向(右)移動 7 單位
   向 y 軸正向(上)移動 6 單位
   可以得到 $(1,1) \Rightarrow a+b=13$



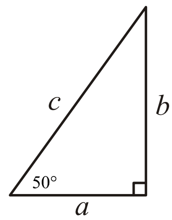
10. A、B 中點在 x 軸上
   $\Rightarrow \dfrac{(a+2b-10)+(5a+4b-2)}{2}=0$
   $\Rightarrow 3a+3b-6=0 \Rightarrow a+b-2=0$
   A、C 中點在 y 軸上
   $\Rightarrow \dfrac{(-2a+7b)+(6a-b)}{2}=0 \Rightarrow 2a+3b=0$
   $\Rightarrow \begin{cases} a+b-2=0 \\ 2a+3b=0 \end{cases} \Rightarrow a=6$，$b=-4 \Rightarrow a+b=2$

11. $\because \sin 2\theta\cos\theta>0 \Rightarrow 2\sin\theta\cos\theta\cos\theta>0$
   $\Rightarrow 2\sin\theta\cos^2\theta>0 \Rightarrow \sin\theta>0 \Rightarrow$ 在一 or 二象限
   $\because \cot\theta\sin\theta<0 \Rightarrow \cot\theta<0 \Rightarrow \theta$ 在二 or 四象限
   $\Rightarrow \theta$ 在第二象限

12. 如右圖
   $\Rightarrow 2r+r\theta=3r\theta \Rightarrow 2r=2r\theta$
   $\Rightarrow \theta=1$(弧度)$=\dfrac{180°}{\pi}$ (六十分制)



13. 如右圖，$\sin(-130°)=-\sin 50°=\dfrac{-b}{c}$
   $\cos(-130°)=-\cos 50°=\dfrac{-a}{c}$
   $\tan(-130°)=\tan 50°=\dfrac{b}{a}$
   $\cot(-130°)=\cot 50°=\dfrac{a}{b}$
   $\because c>b>a>0 \Rightarrow \dfrac{b}{c}>\dfrac{a}{c}$
   $\Rightarrow \sin(-130°)=-\sin 50°=\dfrac{-b}{c}$ 最小



14. 只要是 $\sin\theta$ 都會有意義

$\csc 1080° = \csc 0° = \dfrac{1}{\sin 0°} = \dfrac{1}{0}$ 沒有意義

$\sec 720° = \sec 0° = \dfrac{1}{\cos 0°} = \dfrac{1}{1} = 1$

$\cot 810° = \cot 90° = 0$

15. $\tan\theta < 0$ ， $\sec\theta = \dfrac{\sqrt{13}}{2}$

$\Rightarrow \sin\theta = \dfrac{-3}{\sqrt{13}}$ ， $\cos\theta = \dfrac{2}{\sqrt{13}}$

$\Rightarrow \cos 2\theta = \cos^2\theta - \sin^2\theta = \dfrac{4}{13} - \dfrac{9}{13} = \dfrac{-5}{13}$

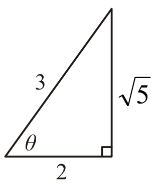
或 $\tan\theta < 0$ ， $\sec\theta = \dfrac{\sqrt{13}}{2} \Rightarrow \tan\theta = \dfrac{-3}{2}$

$\cos 2\theta = \dfrac{1-\tan^2\theta}{1+\tan^2\theta} = \dfrac{1-\frac{9}{4}}{1+\frac{9}{4}} = \dfrac{-5}{13}$

16. $\because \cot\theta = \dfrac{2}{\sqrt{5}} \Rightarrow \sin\theta = \dfrac{\sqrt{5}}{3}$

$\tan\theta = \dfrac{\sqrt{5}}{2}$ ， $\cos\theta = \dfrac{2}{3}$

$\dfrac{\sin\theta \times \tan\theta}{\cos\theta + \sec\theta} = \dfrac{\frac{\sqrt{5}}{3} \times \frac{\sqrt{5}}{2}}{\frac{2}{3} + \frac{3}{2}} = \dfrac{\frac{5}{6}}{\frac{13}{6}} = \dfrac{5}{13}$



17. 設 $B(x_2, y_2)$ 、 $C(x_3, y_3)$

重心坐標爲 $(5,3) = (\dfrac{1+x_2+x_3}{3}, \dfrac{-3+y_2+y_3}{3})$

$\Rightarrow \dfrac{1+x_2+x_3}{3} = 5$ ， $\dfrac{-3+y_2+y_3}{3} = 3$

$\Rightarrow x_2 + x_3 = 14$ ， $y_2 + y_3 = 12$

$\Rightarrow \dfrac{x_2+x_3}{2} = 7$ ， $\dfrac{y_2+y_3}{2} = 6$

$\Rightarrow B$ 、C 中點坐標爲(7,6)

18. $\tan\theta + \cot\theta = \dfrac{-32}{7} \Rightarrow \dfrac{1}{\sin\theta\cos\theta} = \dfrac{-32}{7}$

$\Rightarrow \sin\theta\cos\theta = \dfrac{-7}{32}$

$(\sin\theta - \cos\theta)^2 = \sin^2\theta - 2\sin\theta\cos\theta + \cos^2\theta$

$= 1 - 2 \times \dfrac{-7}{32} = \dfrac{46}{32} = \dfrac{23}{16} \Rightarrow \sin\theta - \cos\theta = \dfrac{\sqrt{23}}{4}$

($\because 90° < \theta < 135° \Rightarrow \sin\theta - \cos\theta > 0$)

19. $0 < \alpha < \dfrac{\pi}{2}$ ， $\cot\alpha = \dfrac{4}{3} \Rightarrow \sin\alpha = \dfrac{3}{5}$ ， $\cos\alpha = \dfrac{4}{5}$

$\pi < \beta < \dfrac{3\pi}{2}$ ， $\tan\beta = \dfrac{12}{5} \Rightarrow \sin\beta = \dfrac{-12}{13}$ ， $\cos\beta = \dfrac{-5}{13}$

$\sin(\alpha+\beta) = \sin\alpha\cos\beta + \cos\alpha\sin\beta$

$= \dfrac{3}{5} \times \dfrac{-5}{13} + \dfrac{4}{5} \times \dfrac{-12}{13} = \dfrac{-63}{65}$

20. $\begin{cases} 3\sin\theta + 6\sqrt{2}\cos\theta - 9 = 0 \\ -\sin\theta + \sqrt{2}\cos\theta - 1 = 0 \end{cases}$

$\Rightarrow \begin{cases} 3\sin\theta + 6\sqrt{2}\cos\theta - 9 = 0 \cdots (1) \\ -9\sin\theta + 9\sqrt{2}\cos\theta - 9 = 0 \cdots (2) \end{cases}$

$(1)-(2)$ ，$12\sin\theta - 3\sqrt{2}\cos\theta = 0 \Rightarrow 12\sin\theta = 3\sqrt{2}\cos\theta$

$\Rightarrow \tan\theta = \dfrac{\sin\theta}{\cos\theta} = \dfrac{3\sqrt{2}}{12} = \dfrac{\sqrt{2}}{4}$

或 $\begin{cases} 3\sin\theta + 6\sqrt{2}\cos\theta - 9 = 0 \\ -\sin\theta + \sqrt{2}\cos\theta - 1 = 0 \end{cases} \Rightarrow \sin\theta = \dfrac{1}{3}$

$\cos\theta = \dfrac{2\sqrt{2}}{3} \Rightarrow \tan\theta = \dfrac{\sin\theta}{\cos\theta} = \dfrac{\frac{1}{3}}{\frac{2\sqrt{2}}{3}} = \dfrac{1}{2\sqrt{2}} = \dfrac{\sqrt{2}}{4}$

21. $\sin\theta + \cos\theta = \dfrac{1}{\sqrt{3}} \Rightarrow \sin^2\theta + \cos^2\theta + 2\sin\theta\cos\theta = \dfrac{1}{3}$

$\Rightarrow \sin\theta\cos\theta = \dfrac{-1}{3}$

$\tan^2\theta + \cot^2\theta = (\tan\theta + \cot\theta)^2 - 2\tan\theta\cot\theta$

$= (\dfrac{1}{\sin\theta\cos\theta})^2 - 2 = (\dfrac{1}{\frac{-1}{3}})^2 - 2 = 7$

22. $\alpha < 0 \Rightarrow \tan\alpha = \dfrac{-1}{2}$

$\tan(360° - \beta) = \dfrac{3}{2} = \tan(-\beta) \Rightarrow \tan\beta = \dfrac{-3}{2}$

$\tan(\alpha - \beta) = \dfrac{\tan\alpha - \tan\beta}{1 + \tan\alpha \cdot \tan\beta} = \dfrac{\frac{-1}{2} - \frac{-3}{2}}{1 + \frac{-1}{2} \cdot \frac{-3}{2}}$

$= \dfrac{1}{\frac{7}{4}} = \dfrac{4}{7} \Rightarrow \cot(\alpha - \beta) = \dfrac{7}{4}$

23. $\cot\theta + \csc\theta = 4 \Rightarrow \dfrac{\cos\theta}{\sin\theta} + \dfrac{1}{\sin\theta} = 4 \Rightarrow \dfrac{\cos\theta + 1}{\sin\theta} = 4$

$\Rightarrow \cos\theta + 1 = 4\sin\theta \Rightarrow \cos^2\theta + 2\cos\theta + 1$

$= 16\sin^2\theta = 16(1 - \cos^2\theta) = 16 - 16\cos^2\theta$

$\Rightarrow 17\cos^2\theta + 2\cos\theta - 15 = 0$

$\Rightarrow (17\cos\theta - 15)(\cos\theta + 1) = 0$

$\Rightarrow \cos\theta = \dfrac{15}{17}$ ， -1 (不合，θ 爲第一象限角)

$\Rightarrow \sin\theta = \dfrac{8}{17} \Rightarrow \sin\theta + \cos\theta = \dfrac{8}{17} + \dfrac{15}{17} = \dfrac{23}{17}$

24. (法 1)

與 x 軸交於 $(m,0)$ 、 $(n,0)$

可以假設此函數頂點坐標爲 $(\dfrac{m+n}{2}, k) = (1, k)$

故 $f(x) = ax^2 + bx + c = a(x-1)^2 + k$

通過 $(1,-27) \Rightarrow f(1) = k = -27$

$(0,-24) \Rightarrow f(0) = a \times 1 + k = a - 27 = -24 \Rightarrow a = 3$

$f(x) = 3(x-1)^2 - 27 = 3x^2 - 6x - 24$

$\Rightarrow a = 3$ 、 $b = -6$ 、 $c = -24$

$\Rightarrow 3a - 2b + c = 3 \times 3 - 2 \times (-6) + (-24) = -3$

(法 2)

通過 $(m,0) \Rightarrow am^2 + bm + c = 0 \cdots (1)$

$(n,0) \Rightarrow an^2 + bn + c = 0 \cdots (2)$

$(1,-27) \Rightarrow a + b + c = -27$

$(0,-24) \Rightarrow c = -24 \Rightarrow a + b = -3 \cdots (3)$

$(1) - (2) \Rightarrow a(m^2 - n^2) + b(m - n) = 0$

$\Rightarrow a(m+n)(m-n) + b(m-n) = 0$

$\Rightarrow a(m+n) + b = 0 \Rightarrow 2a + b = 0 \cdots (4)$

$(4) - (3) \Rightarrow a = 3 \Rightarrow b = -6$

$\Rightarrow 3a - 2b + c = 3 \times 3 - 2 \times (-6) + (-24) = -3$

25. $2(x+6)^2 - (x+3)^2 + 20 = \sqrt{2}[\sin(\frac{\pi}{6} + \theta) + \cos(\frac{\pi}{6} + \theta)]$

$\Rightarrow x^2 + 18x + 83$

$= \sqrt{2} \times \sqrt{2}[\sin(\frac{\pi}{6} + \theta) \times \frac{1}{\sqrt{2}} + \cos(\frac{\pi}{6} + \theta) \times \frac{1}{\sqrt{2}}]$

$\Rightarrow (x+9)^2 + 2$

$= \sqrt{2} \times \sqrt{2}[\sin(\frac{\pi}{6} + \theta) \times \cos\frac{\pi}{4} + \cos(\frac{\pi}{6} + \theta) \times \sin\frac{\pi}{4}]$

$\Rightarrow (x+9)^2 + 2 = 2\sin(\frac{5\pi}{12} + \theta)$

$\because (x+9)^2 + 2 \geq 2$ ， $-2 \leq 2\sin(\frac{5\pi}{12} + \theta) \leq 2$

$\Rightarrow (x+9)^2 + 2 = 2 \Rightarrow x = -9$