九十八學年四技二專第一次聯合模擬考試
共同考科　數學(C)卷　詳解

數學(C)卷 98-1-C

1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	17	18	19	20	21	22	23	24	25
B	A	C	D	C	A	B	C	D	A	D	C	D	B	C	A	B	D	C	B	B	D	A	A	A

1. $\begin{cases}2a+3b-10=0\\a-b+5=0\end{cases}\Rightarrow a=-1$ 、 $b=4$

   $P(-1,4)\in II$

2. $\dfrac{2x-3}{4x+1}=-1\Rightarrow x=\dfrac{1}{3}$

   代入原式，則 $f(-1)=\dfrac{\frac{1}{3}-1}{2\times\frac{1}{3}+1}=\dfrac{-\frac{2}{3}}{\frac{5}{3}}=-\dfrac{2}{5}$

3. $f(x)=-(x-1)^2+7$ ；∵領導係數小於 0

   則有最高點(最大值)，當 $x=1$時，有最大值 7

   ∴ $p=1$ ， $q=7\Rightarrow 3p+q=10$

4. $x=\dfrac{2\times1-1\times5}{3}=-1$ ， $y=\dfrac{2\times2+1\times5}{3}=3$

5. (1) 過 C 作 $\overline{CD}\perp\overline{AB}$ 交 $\overline{AB}$ 於 D

   ∵ $\angle A=60^\circ\Rightarrow\dfrac{2}{\sqrt{3}}=\dfrac{12}{\overline{CD}}\Rightarrow\overline{CD}=6\sqrt{3}$

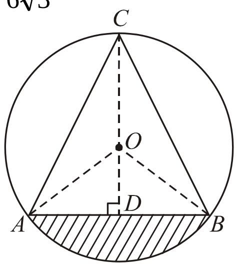
   又 O 爲正 ΔABC 的外心

   也是重心

   由重心性質知

   $\overline{OC}=\dfrac{2}{3}\overline{CD}=\dfrac{2}{3}\times6\sqrt{3}=4\sqrt{3}$

   $\overline{OD}=\dfrac{1}{3}\overline{CD}=\dfrac{1}{3}\times6\sqrt{3}=2\sqrt{3}$



   (2) 連接 $\overline{OA}\Rightarrow\angle AOB=120^\circ=\dfrac{2}{3}\pi$

   (3) 弓形面積(斜線區域面積)

   $=$(一圓心角爲 $\dfrac{2}{3}\pi$之扇形)$-$(一等腰三角形ΔAOB)

   $=\dfrac{1}{2}\times(4\sqrt{3})^2\times\dfrac{2}{3}\pi-\dfrac{1}{2}\times12\times2\sqrt{3}=16\pi-12\sqrt{3}$

6. 由根與係數的關係得知

   $\begin{cases}\cot\alpha+\cot\beta=3\\\cot\alpha\times\cot\beta=2\end{cases}\Rightarrow\begin{cases}\dfrac{1}{\tan\alpha}+\dfrac{1}{\tan\beta}=3\\\tan\alpha\times\tan\beta=\dfrac{1}{2}\end{cases}$

   $\Rightarrow\begin{cases}\tan\alpha+\tan\beta=\dfrac{3}{2}\\\tan\alpha\times\tan\beta=\dfrac{1}{2}\end{cases}$

   $\therefore\tan(\alpha+\beta)=\dfrac{\tan\alpha+\tan\beta}{1-\tan\alpha\times\tan\beta}=3$

   由三角恆等式之平方關係知

   $\cos^2(\alpha+\beta)=\dfrac{1}{\sec^2(\alpha+\beta)}=\dfrac{1}{1+\tan^2(\alpha+\beta)}$

   $=\dfrac{1}{10}\Rightarrow 10\cos^2(\alpha+\beta)=1$，因此

   原式$\Rightarrow 10\cos^2(\alpha+\beta)+\cos^2(\alpha+\beta)+\sin^2(\alpha+\beta)$

   $=1+1=2$

7. $\tan\theta+\cot\theta=\dfrac{\sin\theta}{\cos\theta}+\dfrac{\cos\theta}{\sin\theta}$

   $=\dfrac{1}{\cos\theta\cdot\sin\theta}=\dfrac{5}{3}\Rightarrow\cos\theta\cdot\sin\theta=\dfrac{3}{5}$

   代回原式得

   $(\sin\theta+\cos\theta)^2=1+2\sin\theta\cdot\cos\theta=1+\dfrac{6}{5}=\dfrac{11}{5}$

8. 由餘弦定理知得 $b^2=a^2+c^2-2ac\cos\theta$

   $\Rightarrow b^2=3^2+5^2-2\times3\times5\times(-\dfrac{1}{2})=49\Rightarrow b=7$

   內接圓半徑 $r=\dfrac{\Delta ABC}{s}=\dfrac{\frac{1}{2}\times3\times5\times\sin120^\circ}{\frac{1}{2}(3+5+7)}=\dfrac{\sqrt{3}}{2}$

   內接圓面積 $\pi r^2=\pi\times(\dfrac{\sqrt{3}}{2})^2=\dfrac{3}{4}\pi$

9. 設三邊長爲 $a=5$ ， $b=6$ ， $c=7$

   依海龍公式： $s=\dfrac{1}{2}(a+b+c)=9$

   $\Delta=\sqrt{s(s-a)(s-b)(s-c)}=\sqrt{9\times(9-5)(9-6)(9-7)}$

   $=\sqrt{9\times4\times3\times2}=6\sqrt{6}$

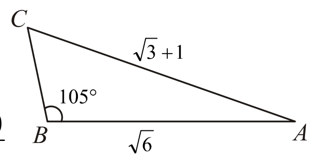
10. $\begin{cases}p=6\times\dfrac{1}{2}\times(\dfrac{12}{6})\times(\dfrac{12}{6})\times\sin60^\circ=6\sqrt{3}\ cm^2\\q=\dfrac{1}{2}\times(\dfrac{12}{3})\times(\dfrac{12}{3})\times\sin60^\circ=4\sqrt{3}\ cm^2\end{cases}$

    $\Rightarrow p+q=10\sqrt{3}$

11. 利用正弦定理得知

    $\dfrac{\overline{AB}}{\sin C}=\dfrac{\overline{AC}}{\sin B}\Rightarrow\dfrac{\sqrt{6}}{\sin C}=\dfrac{\sqrt{3}+1}{\sin105^\circ}$

    $\Rightarrow\dfrac{\sqrt{6}}{\sin C}=\dfrac{\sqrt{3}+1}{\frac{\sqrt{6}+\sqrt{2}}{4}}$



    $\Rightarrow(\sqrt{3}+1)\sin C=\dfrac{2\sqrt{3}(\sqrt{3}+1)}{4}$

    $\therefore\sin C=\dfrac{\sqrt{3}}{2}$，故 $\angle C=60^\circ$ 或 120°

但 $\overline{AC} > \overline{AB}$，$\therefore \angle B > \angle C$，$\therefore \angle C = 60°$

12. $\dfrac{a}{\sin A} = 2R \Rightarrow \dfrac{8}{\frac{4}{5}} = 2R \Rightarrow R = 5$

13. ΔDCA 中，$\angle A = 45° \Rightarrow \overline{AC} = \overline{DC} = 5$

ΔBCA 中，$\angle A = 60° \Rightarrow \tan 60° = \tan A = \dfrac{\overline{BC}}{\overline{AC}}$

$\Rightarrow \dfrac{5+\overline{BD}}{5} = \dfrac{\sqrt{3}}{1} \Rightarrow \overline{BD} = 5(\sqrt{3}-1)$

14. $\because 0° < \theta < 90°$，$\therefore \theta \in I$

$\cos\theta = \dfrac{1}{3}$，代入原式 $\dfrac{\sin\theta}{1+\cos\theta} = \dfrac{\frac{2\sqrt{2}}{3}}{1+\frac{1}{3}} = \dfrac{\sqrt{2}}{2}$

15. $\cos(-110°) = \cos 110° = p \Rightarrow \cos 70° = -p$

$\therefore \tan 70° = -\dfrac{\sqrt{1-p^2}}{p}$

16. $v = \dfrac{\sin(\pi+\theta)\cdot\tan^2(\pi+\theta)}{\cos(\frac{3}{2}\pi+\theta)} - \dfrac{\sin(\frac{3}{2}\pi-\theta)}{\sin(\frac{\pi}{2}-\theta)\cdot\cos^2(\pi-\theta)}$

$= \dfrac{(-\sin\theta)\cdot\tan^2\theta}{\sin\theta} - \dfrac{(-\cos\theta)}{\cos\theta\cdot(-\cos\theta)^2}$

$= -\tan^2\theta + \dfrac{1}{\cos^2\theta} = -\dfrac{\sin^2\theta}{\cos^2\theta} + \dfrac{1}{\cos^2\theta} = 1$

17. 使用函數疊合性知 $-\sqrt{a^2+b^2}\,(\min imum)$

$\le a\sin x + b\cos x \le \sqrt{a^2+b^2}\,(Maximum)$

$-\sqrt{24^2+(-7)^2} - 13 \le 24\sin x + (-7)\cos x - 13$

$\le \sqrt{24^2+(-7)^2} - 13$，$\therefore -38 \le f(x) \le 12$

$\Rightarrow M = 12$，$m = -38 \Rightarrow M - m = 50$

18. 原式 $= \tan\dfrac{\pi}{6} \times \csc\dfrac{\pi}{4} + \sin\dfrac{5\pi}{4} \times \sec\dfrac{\pi}{3}$

$= \dfrac{\sqrt{3}}{3} \times \sqrt{2} + (-\dfrac{\sqrt{2}}{2}) \times 2 = \dfrac{\sqrt{6}-3\sqrt{2}}{3}$

19. 由餘弦定理知 $\cos B = \dfrac{a^2+c^2-b^2}{2ac} = \dfrac{64+25-49}{80} = \dfrac{1}{2}$

$\Rightarrow \angle B = 60°$

20. $\overrightarrow{AB} = (-4,3)$，$\overrightarrow{BC} = (-4,-1)$，$\overrightarrow{CD} = (2,-1)$

$\overrightarrow{OP} = \overrightarrow{AB} + \overrightarrow{BC} - \overrightarrow{CD} = (-10,3)$，$\therefore P(-10,3)$

21. $4(3\vec{a}+\vec{u}) - 3(5\vec{u}-\vec{b}) = 0 \Rightarrow 12\vec{a} + 4\vec{u} - 15\vec{u} + 3\vec{b} = 0$

$\Rightarrow 11\vec{u} = 12\vec{a} + 3\vec{b}$，$11\vec{u} = 12(-2,4) + 3(5,-8) = (-9,24)$

$\Rightarrow \therefore \vec{u} = (-\dfrac{9}{11}, \dfrac{24}{11})$

22. $\vec{a} + t\vec{b} = (-2t+4, t+3)$，$\vec{a} - \vec{b} = (6,2)$

$(\vec{a}+t\vec{b}) \perp (\vec{a}-\vec{b}) \Rightarrow (\vec{a}+t\vec{b})\cdot(\vec{a}-\vec{b}) = 0$

$\Rightarrow (-2t+4, t+3)\cdot(6,2) = 0 \Rightarrow -10t + 30 = 0$

$\therefore t = 3$

23. 依柯西不等式得

$(x^2+y^2)(5^2+4^2) \ge (5x+4y)^2 \Rightarrow (x^2+y^2) \ge 41$

則最小値爲 41

24. $|3\vec{a}+2\vec{b}|^2 = 9|\vec{a}|^2 + 12(\vec{a}\cdot\vec{b}) + 4|\vec{b}|^2 = 28$

$28 = 40 + 12\vec{a}\cdot\vec{b} \Rightarrow \vec{a}\cdot\vec{b} = -1$

$\Rightarrow \cos\theta = \dfrac{\vec{a}\cdot\vec{b}}{|\vec{a}||\vec{b}|} = -\dfrac{1}{2}$，$\therefore \theta = 120°$

25. $\because (\sin\theta + \cos\theta)^2 = 1 + 2\sin\theta\times\cos\theta$

$\therefore (\sqrt{2})^2 = 1 + 2\sin\theta\times\cos\theta \Rightarrow 2\sin\theta\times\cos\theta = 1$

$\Rightarrow \sin\theta\times\cos\theta = \dfrac{1}{2}$

又 $\vec{a} - \vec{b} = (\sin\theta,1) - (\cos\theta,2) = (\sin\theta - \cos\theta, -1)$

則 $|\vec{a}-\vec{b}| = \sqrt{(\sin\theta-\cos\theta)^2 + (-1)^2}$

$= \sqrt{1 - 2\sin\theta\cdot\cos\theta + 1} = \sqrt{1-1+1} = 1$