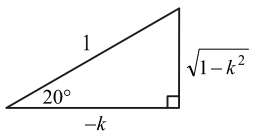
九十八學年四技二專第三次聯合模擬考試
共同考科　數學(C)卷　詳解

數學(C)卷　98-3-C

1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	17	18	19	20	21	22	23	24	25
D	D	B	C	B	A	A	C	D	D	B	C	B	B	A	A	C	D	B	C	A	C	D	D	B

1. 由已知 $f(3+t)=f(3-t)$ 可知拋物線的對稱軸方程式爲 $x=\dfrac{3+t+3-t}{2}=3$，$(3,f(3))$ 爲最低點
$\therefore f(3)$ 爲最小值，而 x 愈接近 3，函數值愈小
$\therefore f(3)<f(2)<f(0)$

2. $\cos 560^\circ=-\cos 20^\circ=k$
$\Rightarrow \cos 20^\circ=-k$
$\sin 860^\circ=\sin 40^\circ$

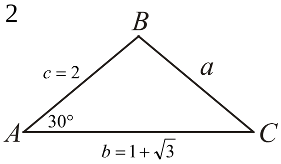


$=2\sin 20^\circ\cos 20^\circ=2(\sqrt{1-k^2})(-k)=-2k\sqrt{1-k^2}$

3. $\because \tan\alpha+\tan\beta=3$，$\tan\alpha\cdot\tan\beta=2$
$\therefore \tan(\alpha+\beta)=\dfrac{\tan\alpha+\tan\beta}{1-\tan\alpha\tan\beta}=\dfrac{3}{1-2}=-3$
$\therefore \sin^2(\alpha+\beta)=(\dfrac{\pm 3}{\sqrt{10}})^2=\dfrac{9}{10}$

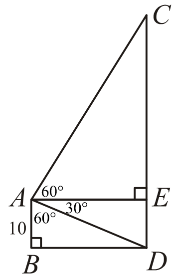
4. (A) 週期爲 $\dfrac{2\pi}{2}=\pi$
(B) $f(x+\pi)=3\cos(2x+2\pi+\dfrac{\pi}{4})$
$=3\cos(2x+\dfrac{\pi}{4})=f(x)$
(C) $\because y$ 的最大值爲 3，y 的最小值爲 -3，$\therefore$和爲 0
(D) $\because f(\dfrac{\pi}{2})=3\cos(\pi+\dfrac{\pi}{4})=-\dfrac{3}{2}\sqrt{2}$
$f(-\dfrac{\pi}{2})=3\cos(-\pi+\dfrac{\pi}{4})=-\dfrac{3}{2}\sqrt{2}$
$\therefore f(\dfrac{\pi}{2})+f(-\dfrac{\pi}{2})=-3\sqrt{2}$

5. 由餘弦定律 $a^2=b^2+c^2-2bc\cos A$
$=(1+\sqrt{3})^2+2^2-2(1+\sqrt{3})\cdot 2\cdot\dfrac{\sqrt{3}}{2}$
$=1+2\sqrt{3}+3+4-2\sqrt{3}-6=2$
$\therefore a=\sqrt{2}=\overline{BC}$



由正弦定律 $\dfrac{a}{\sin A}=\dfrac{c}{\sin C}\Rightarrow\dfrac{\sqrt{2}}{\frac{1}{2}}=\dfrac{2}{\sin C}\Rightarrow \sin C=\dfrac{1}{\sqrt{2}}$
$\Rightarrow \angle C=45^\circ$ ($\because$邊長 b 最大，$\therefore \angle C=135^\circ$ 不合)
$\therefore \angle B=180^\circ-45^\circ-30^\circ=105^\circ$
$\Delta ABC=\dfrac{1}{2}bc\sin A=\dfrac{1}{2}(1+\sqrt{3})\times 2\times\dfrac{1}{2}=\dfrac{1}{2}+\dfrac{\sqrt{3}}{2}$

6. $\because \overline{AB}=10$
$\therefore \overline{BD}=10\sqrt{3}=\overline{AE}$
而 $\overline{CE}=10\sqrt{3}\times\sqrt{3}=30$
$\therefore \overline{CD}=\overline{CE}+\overline{ED}$
$=30+10=40$ (m)



7. $\overrightarrow{AB}\cdot\overrightarrow{BC}=(-\overrightarrow{BA})\cdot\overrightarrow{BC}=-(\overrightarrow{BA}\cdot\overrightarrow{BC})=-|\overrightarrow{BA}||\overrightarrow{BC}|\cos B$
$=-(6\times 7\times\dfrac{6^2+7^2-5^2}{2\times 6\times 7})=-30$

8. $\vec{a}$ 在 $\vec{b}$ 上之正射影長爲 $\dfrac{|\vec{a}\cdot\vec{b}|}{|\vec{b}|}=\dfrac{|7\times(-4)+1\times 3|}{\sqrt{(-4)^2+3^2}}$
$=\dfrac{25}{5}=5$

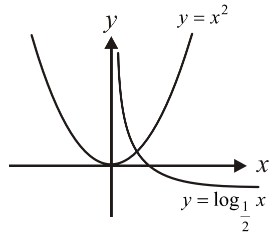
9. 設 $f(x)=(x^2+1)(x+k)+(x+2)$
由餘式定理知 $f(1)=(1+1)(1+k)+3=3$，$\therefore k=-1$
$\therefore f(x)=(x^2+1)(x-1)+(x+2)=x^3-x^2+2x+1$
$\therefore a=-1$、$b=2$、$c=1$，$f(-1)=-1-1-2+1=-3$
$f(0)=1$

10. 由根與係數知 $\alpha+\beta=-6$，$\alpha\beta=4$
$\therefore \alpha<0$，$\beta<0$，$\therefore \sqrt{\alpha}\times\sqrt{\beta}=-\sqrt{\alpha\beta}=-2$
$\dfrac{\sqrt{\beta}}{\sqrt{\alpha}}+\dfrac{\sqrt{\alpha}}{\sqrt{\beta}}=\dfrac{(\sqrt{\beta})^2+(\sqrt{\alpha})^2}{\sqrt{\alpha}\cdot\sqrt{\beta}}=\dfrac{\alpha+\beta}{-\sqrt{\alpha\beta}}=\dfrac{-6}{-2}=3$

11. 由餘弦定律知 $\overline{AB}^2=1^2+3^2-2\times 1\times 3\times\cos 120^\circ=13$
$\therefore \overline{AB}=\sqrt{13}$

12. $\dfrac{w-1}{w}=1+i\Rightarrow 1-\dfrac{1}{w}=1+i\Rightarrow\dfrac{1}{w}=-i\Rightarrow w=i$
$\therefore |w-1|=|-1+i|=\sqrt{2}$

13. $\because$由 $y=x^2$ 和 $y=\log_{\frac{1}{2}}x$ 的圖形知有一個交點
$\therefore$方程式 $x^2=\log_{\frac{1}{2}}x$ 只有一個實數解



14. (a) $\because \sqrt{2}=2^{\frac{1}{2}}$，$\sqrt[3]{2}=2^{\frac{1}{3}}$ 且 $\dfrac{1}{2}>\dfrac{1}{3}$
$\therefore \sqrt{2}>\sqrt[3]{2}$，$\therefore \sqrt{2}-\sqrt[3]{2}>0$

(b) $\because \log_3 2 < \log_3 3 = 1$，$\therefore (\log_3 2) - 1 < 0$

(c) $\because \log_2 3 > \log_2 2 = 1$，$\therefore (\log_2 3) - 1 > 0$

(d) $\log_{\frac{1}{3}} \frac{1}{2} = \log_3 2 > 0$

(e) $\log_{\frac{1}{2}} 3 = -\log_2 3 < 0$，$\therefore$(a)(c)(d)爲正數

15. $8^{-\frac{2}{3}} = (2^3)^{-\frac{2}{3}} = 2^{-2} = \frac{1}{4}$，$9^{\log_3 2} = 9^{\log_9 4} = 4$

$\log_3 27 = 3$，$\log_4 \sqrt{2} = \log_{2^2} 2^{\frac{1}{2}} = \frac{1}{4}$

$\therefore$所求$= \frac{1}{4} + 4 + 3 - \frac{1}{4} = 7$

16. 分群$(\frac{1}{1}), (\frac{2}{1}, \frac{1}{2}), (\frac{3}{1}, \frac{2}{2}, \frac{1}{3}), (\frac{4}{1}, \frac{3}{2}, \frac{2}{3}, \frac{1}{4}), \cdots\cdots$

$\because 1 + 2 + 3 + \cdots\cdots + 13 = 91 < 99$

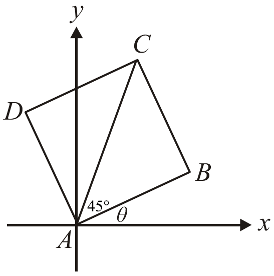
$1 + 2 + 3 + \cdots\cdots + 14 = 105 > 99$

$\therefore$第 99 項爲第 14 群第 8 項即爲$\frac{7}{8}$

17. $\sum_{k=0}^{\infty} (\frac{3^k + 1}{5^k}) = \sum_{k=0}^{\infty} (\frac{3}{5})^k + \sum_{k=0}^{\infty} (\frac{1}{5})^k = \frac{1}{1 - \frac{3}{5}} + \frac{1}{1 - \frac{1}{5}}$

$= \frac{5}{2} + \frac{5}{4} = \frac{15}{4}$

18. 如右圖所示



令θ表$\overline{AB}$之斜角

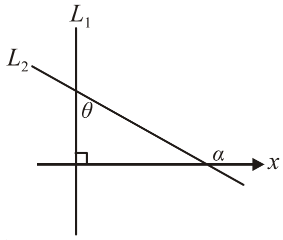
$\because \tan(\theta + 45°) = 3$

$\Rightarrow \frac{\tan\theta + 1}{1 - \tan\theta} = 3$

$\Rightarrow \tan\theta + 1 = 3 - 3\tan\theta$

$\Rightarrow \tan\theta = \frac{1}{2}$，$\therefore \overline{AB}$斜率爲$\frac{1}{2}$

19. L_1之斜角爲$90°$



$\because L_2$之斜率爲$-\frac{1}{\sqrt{3}} = \tan\alpha$

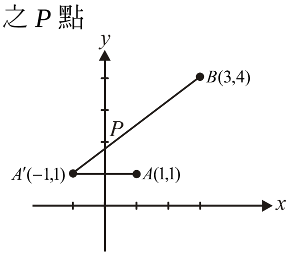
$\therefore$直線L_2之斜角α爲$150°$

$\therefore \theta = 150° - 90° = 60°$

$\cos 2\theta = \cos 120° = -\frac{1}{2}$

20. 取A點對稱於y軸的點$A'(-1,1)$

連$\overline{A'B}$和y軸之交點即爲所求之P點



$\overline{A'B}$：$y - 1 = \frac{4-1}{3-(-1)}(x+1)$

$\Rightarrow y - 1 = \frac{3}{4}(x+1)$

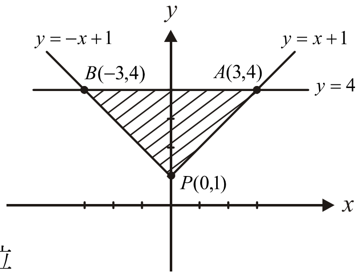
令$x = 0$代入得$y = \frac{7}{4}$

$\therefore P(0, \frac{7}{4}) = (m, n)$，$\overline{PA} + \overline{PB}$之最小值

即爲$\overline{A'B} = \sqrt{[3-(-1)]^2 + (4-1)^2} = 5 = k$

$\therefore m + 4n + k = 0 + 4 \times \frac{7}{4} + 5 = 12$

21. 聯立不等式



$\begin{cases} y \geq |x| + 1 \\ y \leq 4 \end{cases}$

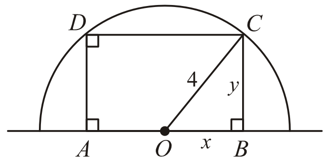
所圍之區域爲圖中

斜線部分ΔABP

ΔABP面積

$= \frac{1}{2} \times 6 \times 3 = 9$平方單位

22. 令$\overline{OB} = x$



$\overline{BC} = y$

$\because x^2 + y^2 = 4^2 = 16$

由算幾不等式知

$\frac{x^2 + y^2}{2} \geq \sqrt{x^2 y^2} \Rightarrow 8 \geq xy \Rightarrow 2xy \leq 16$

$\therefore ABCD$ 最大面積爲 16 平方公分

23. 圓方程式$2x^2 + 2y^2 - 8x + 12y + 8 = 0$

$(\div 2) \Rightarrow x^2 + y^2 - 4x + 6y + 4 = 0$

$\Rightarrow (x-2)^2 + (y+3)^2 = 9$

$\therefore$圓心$(h, k) = (2, -3)$，$r = 3$，$\therefore h + k + r = 2$

24. $\because P(m, n)$在$y^2 - x = 0$上，$\therefore m = n^2$

$P(m, n)$到L：$x + 2y + 6 = 0$之距離

$k = \frac{|m + 2n + 6|}{\sqrt{1^2 + 2^2}} = \frac{|n^2 + 2n + 6|}{\sqrt{5}} = \frac{|(n+1)^2 + 5|}{\sqrt{5}}$

當$n = -1$時k有最小值$\frac{5}{\sqrt{5}} = \sqrt{5}$

$\therefore P(1, -1) = (m, n)$時$k$最小值爲$\sqrt{5}$

$\therefore k^2 + m^2 - n^2 = 5 + 1 - 1 = 5$

25. $\frac{y^2}{25} - \frac{(x-1)^2}{144} = 1 \Rightarrow a = 5$、$b = 12$

P爲雙曲線上一點，由定義知$\left| \overline{PQ_1} - \overline{PQ_2} \right| = 2a = 10$