九十八學年四技二專第四次聯合模擬考試
共同考科　數學(C)卷　詳解

數學(C)卷 98-4-C

1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	17	18	19	20	21	22	23	24	25
C	A	A	B	C	B	C	D	A	C	D	D	B	A	B	B	D	D	C	D	A	B	A	C	B

1. 設 $P(x,y)$，由 $\overline{PA}^2=\overline{PB}^2=\overline{PC}^2$

$\Rightarrow (x-6)^2+(y-2)^2=x^2+(y+6)^2=(x+1)^2+(y-1)^2$

$\Rightarrow \begin{cases}3x+4y=1\\2x-14y=34\end{cases} \Rightarrow x=3，y=-2$，$\therefore P(3,-2)$

2. $\sin^3\theta+\cos^3\theta$

$=(\sin\theta+\cos\theta)(\sin^2\theta-\sin\theta\cdot\cos\theta+\cos^2\theta)$

$\Rightarrow 1=(\sin\theta+\cos\theta)(1-\sin\theta\cos\theta)$ ……①

令 $t=\sin\theta+\cos\theta$，$\therefore \sin\theta\cos\theta=\dfrac{t^2-1}{2}$ 代入①

$\Rightarrow t^3-3t+2=0\Rightarrow (t-1)(t^2+t-2)=0$

$\Rightarrow (t-1)^2(t+2)=0$

$\because \theta$ 為銳角 $\Rightarrow t=1\Rightarrow \sin\theta+\cos\theta=1$

3. $4x^2+4x-3=0\Rightarrow (2x+3)(2x-1)=0$

$\therefore x=-\dfrac{3}{2}$ 或 $\dfrac{1}{2}$，$\because -1\le\sin\theta\le 1$，$\therefore \sin\theta=\dfrac{1}{2}$

$\cos 2\theta=1-2\sin^2\theta=1-2(\dfrac{1}{2})^2=\dfrac{1}{2}$

4. $f(x)=2\cos(\dfrac{\pi}{3}-x)-2\cos x-3$

$=2(\cos\dfrac{\pi}{3}\cos x+\sin\dfrac{\pi}{3}\sin x)-2\cos x-3$

$=\sqrt{3}\sin x-\cos x-3$

$\Rightarrow Max=\sqrt{(\sqrt{3})^2+(-1)^2}-3=-1$

5. ΔAEG 中，餘弦定理：

$\overline{EG}^2=\overline{EA}^2+\overline{AG}^2-2\overline{EA}\cdot\overline{AG}\cdot\cos\angle EAG$

$=\overline{EA}^2+\overline{AG}^2-2\overline{EA}\cdot\overline{AG}\cdot\cos(360°-90°-90°-\angle BAC)$

$=\overline{EA}^2+\overline{AG}^2-2\overline{EA}\cdot\overline{AG}\cdot\cos(180°-\angle BAC)$

$=\overline{EA}^2+\overline{AG}^2+2\overline{EA}\cdot\overline{AG}\cdot\cos\angle BAC$

$=7^2+9^2+2\times7\times9\times\dfrac{7^2+9^2-8^2}{2\times7\times9}=196$，$\therefore \overline{EG}=14$

6. 由除法原理：

$2x^3+ax^2+x-1=(2x^2+x+1)(x+b)+2x+1$

$=2x^3+(2b+1)x^2+(b+3)x+(b+1)$

比較係數可知 $\begin{cases}a=2b+1\\1=b+3\\-1=b+1\end{cases} \Rightarrow a=-3，b=-2$

$\therefore a-b=-3-(-2)=-1$

7. $f(9)=15\Rightarrow f(x)\div(x-9)$ 的餘式為 15

$$\begin{array}{rrrrrr|l} 3 & -26 & -11 & +19 & +\quad k & +\quad 6 & 9\\ & +27 & +\ 9 & -18 & +\quad 9 & +(9k+81) & \\ \hline 3 & +\ 1 & -\ 2 & +\ 1 & +(k+9) & +(9k+87) & \end{array}$$

$\therefore 9k+87=15$，$\therefore k=-8$

8. $a=3^{555}=(3^5)^{111}=(243)^{111}$

$b=4^{444}=(4^4)^{111}=(256)^{111}$

$c=6^{333}=(6^3)^{111}=(216)^{111}$

$\because 256>243>216$，$\therefore b>a>c$

9. 由根與係數關係：

$\log_3\alpha+\log_3\beta=1\Rightarrow \log_3\alpha\beta=1\Rightarrow \alpha\beta=3$

10. $\overrightarrow{AB}\cdot\overrightarrow{AC}=|\overrightarrow{AB}||\overrightarrow{AC}|\cos A=3\cdot5\cdot\dfrac{3^2+5^2-7^2}{2\cdot3\cdot5}=-\dfrac{15}{2}$

11. $(\sin15°+i\cos15°)^{10}=(\cos75°+i\sin75°)^{10}$

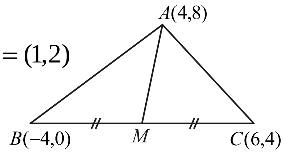
$=\cos750°+i\sin750°=\cos30°+i\sin30°=\dfrac{\sqrt{3}}{2}+\dfrac{1}{2}i$

12. 設首項為 a，公差為 d

$\Rightarrow \begin{cases}a+(a+2d)+(a+4d)+(a+6d)+(a+8d)=25\\(a+d)+(a+3d)+(a+5d)+(a+7d)+(a+9d)=45\end{cases}$

$\Rightarrow \begin{cases}5a+20d=25\\5a+25d=45\end{cases}\Rightarrow d=4$

13. 依題意：作圖如右，$M=\dfrac{B+C}{2}=(1,2)$



$\because m_{\overline{AM}}=\dfrac{8-2}{4-1}=2$

$\therefore \overleftrightarrow{AM}$：$y-2=2(x-1)\Rightarrow 2x-y=0$

14. $m_{\overline{CD}}>m_{\overline{AE}}>m_{\overline{BC}}>m_{\overline{DE}}>m_{\overline{AB}}$

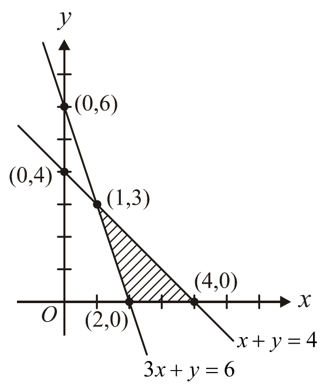
15. 柯西不等式：$[(2a)^2+(b)^2][(\dfrac{1}{2})^2+(1)^2]\ge(a+b)^2$

$\Rightarrow 20\cdot\dfrac{5}{4}\ge(a+b)^2\Rightarrow (a+b)^2\le25\Rightarrow |a+b|\le5$

$\Rightarrow -5\le a+b\le5$，$\therefore Max=5$

16. $\begin{cases}3x+y\ge6\\x+y\le4\\y\ge0\end{cases}$，作圖如下

(x,y)	$x-2y$	
(2,0)	2	
(4,0)	4	→Max
(1,3)	−5	



17. 當直線與圓心距離愈近時，所截的弦愈長

C：$(x-1)^2+(y+2)^2=9\Rightarrow$ 圓心 $M(1,-2)$，半徑 $r=3$

令 L_1：x 軸，L_2：$x+y=1$，L_3：$2x+y=5$

L_4：$3x-4y=7$

(A) $d(M,L_1)=\left|-2\right|=2$

(B) $d(M,L_2)=\dfrac{\left|1+(-2)-1\right|}{\sqrt{1^2+1^2}}=\dfrac{2}{\sqrt{2}}=\sqrt{2}$

(C) $d(M,L_3)=\dfrac{\left|2\times1+1\times(-2)-5\right|}{\sqrt{2^2+1^2}}=\dfrac{5}{\sqrt{5}}=\sqrt{5}$

(D) $d(M,L_4)=\dfrac{\left|3\times1-4\times(-2)-7\right|}{\sqrt{3^2+(-4)^2}}=\dfrac{4}{5}$

18. ∵長軸平行 y 軸，短軸一個頂點爲(0,4)

且其中一焦點爲(4,0)

∴可畫圖如右

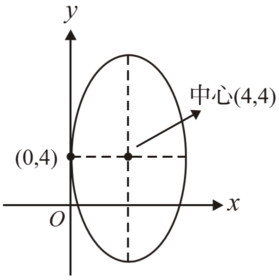
∴中心(4,4)

∴ $c=4$，$b=4$

$a^2=b^2+c^2=16+16=32$

$\Rightarrow a=4\sqrt{2}$

∴長軸的長度 $=2a=8\sqrt{2}$



19. 甲　乙

②　②　③

$\Rightarrow$ 所求 $=C_2^7\cdot C_2^5\cdot C_3^3=210$

20. $C_3^6\cdot C_1^5\ +\ C_2^6\cdot C_2^5\ =250$

(3 男 1 女)(2 男 2 女)

21. ∵和爲奇數必爲一奇一偶

∴所求 $=\dfrac{C_1^2\cdot C_1^2+C_1^2\cdot C_1^2}{C_1^4\cdot C_1^4}=\dfrac{1}{2}$

22. ∵ $f'(12)=\lim\limits_{x\to12}\dfrac{f(x)-f(12)}{x-12}$，其中 $f(12)=0$

∴ $f'(12)=\lim\limits_{x\to12}\dfrac{(x-12)(x-13)^8(x-14)^9}{x-12}$

$=(12-13)^8(12-14)^9=-512$

23. ∵L 與 $2x-6y+1=0$ 垂直，∴ $m_L=-3$

設 $y=f(x)=x^3+3x^2-2\Rightarrow f'(x)=3x^2+6x$

令 $3x^2+6x=-3\Rightarrow x^2+2x+1=0$

$\Rightarrow(x+1)^2=0\Rightarrow x=-1$，∴切點爲 $(-1,0)$

$\Rightarrow L$ 方程式爲 $y-0=(-3)(x+1)\Rightarrow 3x+y+3=0$

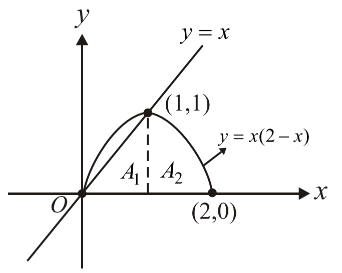
24. $\left|x^2-1\right|=\begin{cases}x^2-1，x\ge1或x\le-1\\-(x^2-1)，-1<x<1\end{cases}$ d

$\int_0^2\left|x^2-1\right|dx=\int_0^1-(x^2-1)dx+\int_1^2(x^2-1)dx$

$=(-\dfrac{1}{3}x^3+x)\Big|_0^1+(\dfrac{1}{3}x^3-x)\Big|_1^2$

$=[(-\dfrac{1}{3}+1)-0]+[(\dfrac{8}{3}-2)-(\dfrac{1}{3}-1)]=\dfrac{2}{3}+\dfrac{4}{3}=2$

25. 依題意作圖如右



$A_1=\dfrac{1}{2}\cdot1\cdot1=\dfrac{1}{2}$

$A_2=\int_1^2 x(2-x)dx$

$=\int_1^2(2x-x^2)dx$

$=(x^2-\dfrac{1}{3}x^3)\Big|_1^2=(4-\dfrac{8}{3})-(1-\dfrac{1}{3})=\dfrac{2}{3}$

∴所求 $=A_1+A_2=\dfrac{1}{2}+\dfrac{2}{3}=\dfrac{7}{6}$