100 學年四技二專第一次聯合模擬考試
共同考科　數學(C)卷　詳解

數學(C)卷 100-1-C

1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	17	18	19	20	21	22	23	24	25
D	C	A	C	D	B	B	C	C	B	B	A	A	C	C	A	B	B	A	D	A	B	D	D	A

1. (A) $f(4)=3\times4-2=10$
(B) $f(0)=f(8)=3\times8-2=22$
(C) $f(10)=f(2)=2^2+5=9$
(D) $f(-2)=f(6)=3\times6-2=16$

2. $\overline{AP}:\overline{AB}=3:5\Rightarrow\overline{AP}:\overline{BP}=3:2$
由內分點公式：
$P(\frac{2\times3+3\times1}{3+2},\frac{2\times2+3\times(-4)}{3+2})\Rightarrow P(\frac{9}{5},-\frac{8}{5})$

3. ΔDEF 的重心亦為 ΔABC 的重心
$\therefore G(\frac{2+3+(-2)}{3},\frac{5+9+4}{3})\Rightarrow G(1,6)$

4. 設 P 點坐標為 $(x,0)$
則 $\overline{PA}^2+\overline{PB}^2$
$=[\sqrt{(x-3)^2+(0-4)^2}]^2+[\sqrt{(x+1)^2+(0-2)^2}]^2$
$=2x^2-4x+30=2(x-1)^2+28$
$\therefore$當 P 點坐標為 $(1,0)$，有最小值 $m=28$

5. $\pi°\doteqdot3.14°$；$\pi°=\frac{\pi}{180}\times\pi=\frac{\pi^2}{180}$ 弳

6. $(\sin\frac{\pi}{8}+\cos\frac{\pi}{8})^2=1+\sin\frac{\pi}{4}=1+\frac{\sqrt{2}}{2}$
$\Rightarrow\sin\frac{\pi}{8}+\cos\frac{\pi}{8}=\pm\sqrt{\frac{2+\sqrt{2}}{2}}=\pm\frac{\sqrt{4+2\sqrt{2}}}{2}$
$\because\sin\frac{\pi}{8}>0$，$\cos\frac{\pi}{8}>0$
$\therefore$取正，$\sin\frac{\pi}{8}+\cos\frac{\pi}{8}=\frac{\sqrt{4+2\sqrt{2}}}{2}$

7. $\frac{2}{(1+\cot\theta)(1-\tan\theta)}=\frac{2}{(1+\frac{1}{\tan\theta})(1-\tan\theta)}$
$=\frac{2\tan\theta}{(\tan\theta+1)(1-\tan\theta)}=\frac{2\tan\theta}{1-\tan^2\theta}=\tan2\theta$

8. $\tan\alpha=\tan[(\alpha+\beta)-\beta]=\frac{\tan(\alpha+\beta)-\tan\beta}{1+\tan(\alpha+\beta)\cdot\tan\beta}$
$=\frac{3-1}{1+3\times1}=\frac{1}{2}$

9. 在終邊上取一點 $(-3,-1)$ 落在第三象限
$r=\sqrt{(-3)^2+(-1)^2}=\sqrt{10}$，則：
(A) $\tan\theta=\frac{y}{x}=\frac{1}{3}$
(B) $\sin\theta=\frac{y}{r}=\frac{-1}{\sqrt{10}}<0$
(C) $\cot\theta=\frac{x}{y}=3$
(D) $\sec\theta=\frac{r}{x}=-\frac{\sqrt{10}}{3}$

10. $\sin^2 1°+\sin^2 2°+\sin^2 3°+\cdots+\sin^2 89°+\sin^2 90°$
$+\sin^2 91°+\cdots+\sin^2 179°+\sin^2 180°$
$=\sin^2 1°+\sin^2 2°+\sin^2 3°+\cdots+\sin^2 89°$
$+1^2+\cos^2 1°+\cdots+\cos^2 89°+0^2$
$=(\sin^2 1°+\cos^2 1°)+(\sin^2 2°+\cos^2 2°)$
$+\cdots+(\sin^2 89°+\cos^2 89°)+1$
$=1+1+1+\cdots+1+1=90$

11. (A) $f(x)$ 的週期為 $\frac{2\pi}{1}=2\pi$
(B) $g(x)$ 的週期為 $\frac{2\pi}{2}=\pi$
(C) $k(x)$ 的週期為 $\frac{\pi}{\frac{1}{2}}=2\pi$
(D) $\ell(x)$ 的週期為 $\frac{2\pi}{1}=2\pi$

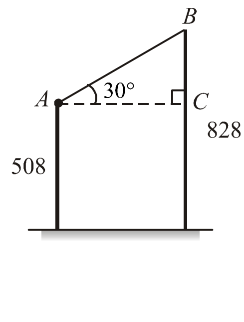
12. $f(x)=\sin^2 x-4\sin x+5=(\sin x-2)^2+1$
$\because 0<x\le2\pi$，$\therefore -1\le\sin x\le1$
故當 $\sin x=-1$ 時，$f(x)$ 有最大值 10

13. 外接圓半徑 $R=\overline{OA}=5$
由正弦定理
$\frac{\overline{AC}}{\sin B}=2R\Rightarrow\frac{6}{\sin B}=10\Rightarrow\sin B=\frac{6}{10}=\frac{3}{5}$

14. $b^2+c^2-a^2=bc\Rightarrow\frac{b^2+c^2-a^2}{2bc}=\frac{1}{2}$
$\Rightarrow\cos A=\frac{1}{2}$ (餘弦定理)，$\therefore\angle A=60°$

15. ΔABC的面積$=\frac{1}{2}\times\overline{AB}\times\overline{AC}\times\sin A$
$=\frac{1}{2}\times12\times5\times\sin30°=15$

16. 如右圖所示
設 101 大樓頂為 A
哈里發塔大樓頂為 B
$\overline{BC}=828-508=320$



$\angle BAC = 30°$，$\angle BCA = 90°$

則 $\dfrac{\overline{BC}}{\overline{AC}} = \tan 30° \Rightarrow \dfrac{320}{\overline{AC}} = \dfrac{1}{\sqrt{3}}$

$\Rightarrow \overline{AC} = 320\sqrt{3}$

17. $\because a = \cos 1 \fallingdotseq \cos 57° > 0$

$b = \cos 2 \fallingdotseq \cos 114° = -\cos 66°$

$c = \cos 3 \fallingdotseq \cos 171° = -\cos 9°$

$\therefore a > b > c$

18. $s = \dfrac{a+b+c}{2} = \dfrac{7+8+9}{2} = 12$

由海龍公式，ΔABC 面積

$\Delta = \sqrt{12 \times (12-7)(12-8)(12-9)} = 12\sqrt{5}$

又 $\Delta = r \times s \Rightarrow 12\sqrt{5} = r \times 12 \Rightarrow r = \sqrt{5}$

19. 設 $\vec{u} = (x, y)$，則 $\begin{cases} x = |\vec{u}| \times \cos 120° = -3 \\ y = |\vec{u}| \times \sin 120° = 3\sqrt{3} \end{cases}$

$\therefore \vec{u} = (-3, 3\sqrt{3})$

20. $(\vec{a} + \vec{b}) \cdot (\vec{a} - \vec{b}) = |\vec{a}|^2 - \vec{a} \cdot \vec{b} + \vec{b} \cdot \vec{a} - |\vec{b}|^2$

$= 4^2 - 5^2 = -9$

21. $\vec{a} + \vec{b} + \vec{c} = \vec{0} \Rightarrow \vec{a} + \vec{b} = -\vec{c} \Rightarrow |\vec{a} + \vec{b}|^2 = |-\vec{c}|^2$

$\Rightarrow |\vec{a}|^2 + 2\vec{a} \cdot \vec{b} + |\vec{b}|^2 = |\vec{c}|^2 \Rightarrow \vec{a} \cdot \vec{b} = \dfrac{|\vec{c}|^2 - |\vec{a}|^2 - |\vec{b}|^2}{2}$

$= \dfrac{4^2 - 2^2 - 3^2}{2} = \dfrac{3}{2}$

22. $\overrightarrow{AB} = (7,1)$、$\overrightarrow{AC} = (3,4)$

則 $\overrightarrow{AB} \cdot \overrightarrow{AC} = |\overrightarrow{AB}| \times |\overrightarrow{AC}| \times \cos A$

$\Rightarrow 21 + 4 = 5\sqrt{2} \times 5 \times \cos A \Rightarrow \cos A = \dfrac{1}{\sqrt{2}} \Rightarrow \angle A = 45°$

23. $(\vec{a} + k\vec{b}) \cdot \vec{a} = 0 \Rightarrow (1+4k) \times 1 + (3+2k) \times 3 = 0$

$\Rightarrow k = -1$

24. $\overrightarrow{AB} = (3,-1)$、$\overrightarrow{AC} = (4,2)$

$|\overrightarrow{AD}| = \overrightarrow{AB}$ 在 $\overrightarrow{AC}$ 上的正射影長

$= \dfrac{|\overrightarrow{AB} \cdot \overrightarrow{AC}|}{|\overrightarrow{AC}|} = \dfrac{10}{\sqrt{20}} = \dfrac{10}{2\sqrt{5}} = \sqrt{5}$

25. $\overrightarrow{AD} = \dfrac{\overrightarrow{AB} \cdot \overrightarrow{AC}}{|\overrightarrow{AC}|^2} \times \overrightarrow{AC} = \dfrac{10}{20} \times (4,2) = (2,1)$

設 $D(x, y)$，則 $\begin{cases} x - 2 = 2 \\ y - 3 = 1 \end{cases} \Rightarrow \begin{cases} x = 4 \\ y = 4 \end{cases}$

$\therefore D(4,4)$